New values of gravitational moments J2 and J4 deduced from helioseismology

Redouane Mecheri, Toufik Abdelatif, Abdanour Irbah, Janine Provost, Gabrielle Berthomieu

New values of gravitational moments J_2 and J_4 deduced from helioseismology

R. Mecheri, T. Abdelatif and A. Irbah
C.R.A.A.G - Observatoire d'Alger *BP 63 Bouzareah, Alger, Algerie.*

J. Provost and G. Berthomieu
Departement Cassini, UMR CNRS 6529 - Observatoire de la Côte d'Azur, BP 4229, 06304 Nice CEDEX4, France.

Abstract. By applying the theory of slowly rotating stars to the Sun, the solar quadrupole and octopole moments J_2 and J_4 were computed using a solar model obtained from *CESAM* stellar evolution code (Morel (1997)) combined with a recent model of solar differential rotation deduced from helioseismology (Corbard *et al.* (2002)). This model takes into account a near-surface radial gradient of rotation which was inferred and quantified from *MDI f-modes* observations by Corbard and Thompson (2002). The effect of this observational near-surface gradient on the theoretical values of the surface parameters J_2, J_4 is investigated. The results show that the octopole moment J_4 is much more sensitive than the quadrupole moment J_2 to the subsurface radial gradient of rotation.

1. Introduction

Several theoretical determinations of the J_2 and J_4 gravitational moments have been undertaken in case of different solar differential rotation laws : (i) only radius dependence (Goldreich and Schubert, 1968; Paternò, Sofia, and Di Mauro, 1996), (ii) quadratic expansion in colatitude cosine terms (Ulrich and Hawkins (1981a and 1981b)), (iii) angular velocity distribution with a slowly latitude variation determined by mean of helioseimology technics (Gough (1982)). More recent determinations are those performed by : (i) Armstrong and Kuhn (1999) using a quadratic expansion rotation law with coefficients obtained by fitting higher resolution helioseismic interior rotation data from MDI (Scherrer *et al.* (1995)), (ii) Godier and Rozelot (1999) and (iii) Roxburgh (2001) using the differential rotation model given by Kosovichev (1996) from *BBSO p-modes* observations. This model takes into account the presence of a constant near-surface radial gradient based on the assumption that the angular momentum is preserved in the supergranulation layer. The aim of the present work is a contribution to J_2 and J_4 determinations using a new analytical model of solar differential rotation provided by Corbard *et al.* (2002) which has a latitudinal dependent profile of the near-surface radial gradient of rotation.

If we consider the Sun as an axial symmetry distribution of matter in rotation, the outer gravitational field ϕ_{out} is expressed as :

$$\phi_{out}(r,\theta) = -\frac{GM_\odot}{r}\left[1 - \sum_{n=1}^{\infty}\left(\frac{R_\odot}{r}\right)^{2n} J_{2n}P_{2n}(\cos\theta)\right] \quad (1)$$

where J_{2n} are the gravitational moments, P_{2n} the Legendre polynomials and r, θ respectively the distance from the Sun centre and the angle to the symmetry axis (colatitude).

Since the solar rotation is slow, it induces small perturbations around the spherical equilibrium. These perturbations can be expanded on Legendre polynomial basis. The distribution of the gravitational potential in the Sun can be written :

$$\phi(r,\theta) = \phi_0(r) + \phi_1(r,\theta) = \phi_0(r) + \sum_{n=1}^{\infty}\phi_{12n}(r)P_{2n}(u) \quad (2)$$

where $u = cos\theta$.

The gravitational moments J_{2n} are obtained assuming the continuity of the gravitational potential at the surface :

$$J_{2n} = \frac{R_\odot}{GM_\odot}\phi_{12n}(R_\odot) \quad (3)$$

The perturbed potential is obtained by linearization of the equation of hydrostatic equilibrium and the Poisson equation, leading to :

$$\begin{aligned}\frac{d^2\phi_{12n}}{dr^2} &+ \frac{2}{r}\frac{d\phi_{12n}}{dr} - (2n(2n+1) + UV)\frac{\phi_{12n}}{r^2} = U[(V+2)B_{2n} + \\ &+ r\frac{dB_{2n}}{dr} + \frac{4n+1}{2}\int_{-1}^{+1}(1-u^2)P_{2n}(u)\Omega(r,u)^2du] \quad (4)\end{aligned}$$

where $U = 4\pi\rho_0 r^3/M_r$, $V = dln\rho_0/dlnr$, M_r is the mass contained in a sphere of radius r and $\Omega(r,u)$ the angular velocity. B_{2n} is given by:

$$B_{2n}(r) = -\frac{1}{2n!}\frac{4n+1}{2^{2n+1}}\int_{-1}^{+1}u\Omega(r,u)^2\frac{d^{2n-1}}{du^{2n-1}}((u^2-1)^{2n})du \quad (5)$$

Equation (3) is integrated with the usual boundary conditions, using U and V provided by a solar model obtained from the *CESAM* stellar evolution code (Morel (1997)) and a rotation law derived from helioseismology.

2. Analytical model of solar differential rotation

We consider the recent Corbard *et al.* (2002) model of solar differential rotation and, for comparison, the Kosovichev one (1996) already used by Roxburgh (2001) and Godier and Rozelot (1999). The Corbard model contains a near-surface radial gradient of rotation inferred from the radial dependence of the *MDI f-modes* observations (Corbard and Thompson (2002)). Two estimations of this gradient have been derived from different sets of modes leading to two rotation models denoted afterward by (a) and (b). As for Kosovichev's model, the surface rotation is forced to surface plasma observations (Snodgrass, 1992).

Figure 1 shows the solar rotation profiles corresponding to these models computed for different latitudes. Kosovichev's model presents a negative constant subsurface radial gradient. The Corbard models have a negative value of the radial gradient at low latitude which is twice smaller than Kosovichev ones. At high latitude, the gradients are positive with larger magnitudes for the Corbard model (b).

We use an analytical expression derived respectively by Corbard *et al.* (2002) and Dikpati *et al.* (2002) for the Corbard rotation laws and for the Kosovichev one. We recall hereafter the full set of equations and parameters they give for these rotation laws :

$$\Omega(r,u) = A_1(r,u) + \Psi_{tac}(r)\left(\Omega_{cz} - \Omega_0 + a_2u^2 + a_4u^4\right) \qquad (6)$$

where

$$\begin{aligned} A_1(r,u) &= \Omega_0 + \Psi_{cz}(r)\left\{\alpha(u)(r-r_{cz})\right\} + \\ &+ \Psi_s(r)\left\{\Omega_{eq} - \Omega_{cz} - \beta(u)(r-R_\odot) - \alpha(u)(r-r_{cz})\right\} \end{aligned} \qquad (7)$$

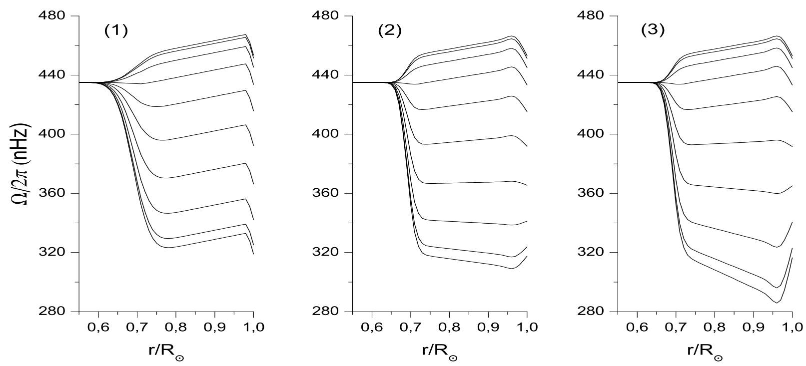

Figure 1. Profiles of the solar rotation from $0.55R_\odot$ to the surface for different latitudes computed each 10° from the Equator (top) to the Pole (bottom).**(1) Model of Kosovichev**. **(2) Model of Corbard (a)**. **(3) Model of Corbard (b)**.

with

$$\alpha(u) = \frac{\Omega_{eq} - \Omega_{cz} + \beta(u)(R_\odot - r_s)}{r_s - r_{cz}}$$

Ω_0, Ω_{eq} and Ω_{cz} are respectively the constant rotation of the radiative interior zone, the equatorial rate at the surface and at the top r_{cz} of the tachocline localized at r_{tac}. The a_2 and a_4 constants describe the latitudinal differential rotation. $\beta(u)$ represents the latitudinal dependance of the rotation radial gradient below the surface down to a radius r_s. This gradient depends on the latitude through the β_0, β_3 and β_6 constants by the Equation : $\beta(u) = \beta_0 + \beta_3 u^3 + \beta_6 u^6$.

The Ψ_x function where x stands for tac, cz or s, models the transition between different gradients. An error function centered at r_x with width ω_x is used for this goal : $\Psi_x(r) = 0.5\,(1 + erf\,[2(r - r_x)/\omega_x])$.
All the rotation laws have the following common parameters : $\Omega_0 = 435nHz$, $\Omega_{eq} = 452.5nHz$, $\Omega_{cz} = 453.5nHz$, $r_{tac} = 0.69R_\odot$, $r_{cz} = 0.71R_\odot$, $a_2 = -61nHz$, $a_4 = -73.5nHz$. The parameters which are different for the three laws are given in Table I.

Table I. The non common parameter values between the rotation models. The β_0, β_3 and β_6 parameters are given in $nHz/R_\odot$.

model	$\omega_{tac}/R_\odot$	$\omega_{cz}/R_\odot$	$\omega_s/R_\odot$	$r_s/R_\odot$	β_0	β_3	β_6
Kosovichev (1996)	0.1	0	0	0.983	891.5	0	0
Corbard (a) (2002)	0.05	0.05	0.05	0.97	437	-214	-503
Corbard (b) (2002)	0.05	0.05	0.05	0.97	437	0	-1445

3. Results and discussion

We present in Table II the computed values of J_2 and J_4 obtained with different solar rotation models described in Section 2 and with an uniform rotation equal to the rotation rate of the solar radiative zone Ω_0, for comparison. Table III gives also some values of J_2 and J_4 presented by other authors.
Our results show that the differential rotation in the convective zone reduces J_2 value of about 0.8% when the Corbard models are considered and about 0.5% in the case of Kosovichev's model. For this last case, our J_2 determination is larger than the value found by Godier and Rozelot (1999) but in agreement with the one obtained by Roxburgh (2001), both using Kosovichev's model. Our values are also in agreement with

Table II. The J_2 and J_4 values corresponding to the different rotation models. $\Omega_0 = 2.733\ \mu rd/s$ is the rotation in the radiative zone.

Model of rotation	$J_2(\times 10^{-7})$	$J_4(\times 10^{-9})$
Uniform rotation (Ω_0)	2.217	0
Kosovichev (1996)	2.205	-4.455
Corbard (a) (2002)	2.201	-5.601
Corbard (b) (2002)	2.198	-4.805

those given by Paternò *et al.* (1996), Pijpers (1998) and Armstrong and Kuhn (1999) (see Table III). All these values deviate from the range obtained by Ulrich and Hawkins (1981a and 1981b) in the case of the rotation law defined as a simple quadratic expansion. The difference between the subsurface radial gradients induces only a small reduction on J_2 values. It is about 0.25% between Kosovichev's model and Corbard's ones. This difference is however about 0.1% between the two Corbard models.

As expected, the effect of the subsurface radial gradient is more important on J_4 gravitational moment. J_4 absolute values obtained using the models of Corbard (a) and (b) are respectively about 20% and 7% larger than the one obtained with Kosovichev's model. The (a) Corbard model increases the J_4 absolute value of about 14% compared to the one obtained from the (b) Corbard model. Our $|J_4|$ value corresponding to Kosovichev's model is in agreement with the one given by Roxburgh (2001) using the same model. However, those obtained from Corbard's models are larger than other values given in Table III. All these values

Table III. Some computed values of J_2 and J_4 obtained by other authors. The large value of Gough (1982) is due to an estimation of the internal rotation deduced from earlier helioseismic observations

Authors	$J_2(\times 10^{-7})$	$J_4(\times 10^{-9})$
Ulrich and Hawkins (1981)	$1.0 < J_2 < 1.5$	$2.0 < \|J_4\| < 5.0$
Gough (1982)	36	-
Paternò *et al.* (1996)	2.22	-
Pijpers (1998)	2.18	-
Godier and Rozelot (1999)	1.6	-
Armstrong and Kuhn (1999)	2.22	-3.84
Roxburgh (2001)	2.206	-4.45

are consistent with the range given by Ulrich and Hawkins (1981a and 1981b) except for the (a) Corbard model.
Rotation induces a distortion of the solar surface which can be roughly related to J_2 through the following quantity often called oblateness :

$$\frac{R_e - R_p}{R_\odot} \approx \frac{3}{2} J_2 + \frac{\Omega_s^2 R_\odot^3}{2GM_\odot} \tag{8}$$

where Ω_s is an effective rotation rate. R_e, R_p and $R_\odot$ are respectively the equatorial, polar and mean solar radius. This formula is strictly valid for an uniform rotation or for a rotation constant on cylinders. For a solar rotation which presents a complex profile not constant on cylinders, Paternò, Sofia, and Di Mauro (1996) proposed an expression of Ω_s derived from the surface rotation $\Omega(R_\odot, u)$. In our case, Ω_s will be the same for the three models since they are built so as to have the same surface rotation. Thus, in this rough description, the effect of different subsurface radial gradients of rotation on the oblateness appears through the modification of the J_2 gravitational moment. It is negligible since the main term in Equation (8) is the surface rotation term. The value of the oblateness found is 9.1×10^{-6}. It is in agreement with observations of Lydon and Sofia (1996) and Rozelot and Rösch (1997) but slightly larger than those of Kuhn *et al.* (1998) and Armstrong and Kuhn (1999). Rozelot, Godier, and Lefebvre (2001) presented new developments taking into account the surface latitudinal differential rotation to link J_2 and J_4 to the solar equatorial and polar radius. To the lowest order, their formula reduces to Equation (8) with another definition of Ω_s. For our surface rotation, their formula leads to an oblateness value equal to 11.4×10^{-6}. We have estimated that the effects of the second terms are of the order of (4/1000) in the case of Kosovichev's model. New constraints on the oblateness and the shape of the solar surface will hopefully be provided by the future *PICARD* microsatellite *CNES-mission* (Thuillier *et al.* (2003)).
In conclusion, the octopole moment J_4 is much more sensitive than the quadrupole moment J_2 to the inclusion of the latitudinal and radial differential rotation in the convective zone and particularly to the subsurface radial gradient of rotation. Indeed, an important subsurface radial gradient at high latitude decreases significantly the value of $|J_4|$ while it does not affect significantly the J_2 value.

Acknowledgements

We thank Thierry Corbard for providing his model of rotation and the referee for his constructive remarks and comments. This work has been performed with support of the French Foreign Affaire Ministry in the framework of scientific cooperation between France and Algeria (contract 00MDU501)

References

Armstrong, J. and Kuhn, J. R.: 1999, *Astrophys. J.* **525**, 533.
Corbard, T. and Thompson, M. J.: 2002, *Solar Phys.* **205**, 211.
Corbard, T., Dikpati, M., Gilman, P.A. and Thompson, M.J.: 2002, in A. Wilson (ed.), *The SOHO 11 Symposium on From Solar Min to Max: Half a Solar Cycle with SOHO*, Davos, Switzerland, ESA SP-508, Noordwijk: ESA Publications Division, ISBN 92-9092-818-2, p. 75.
Dikpati, M., Corbard, T., Thompson, M. J. and Gilman, P. A.: 2002, *Astrophys. J.* **575**, 41.
Godier, S. and Rozelot, M. J.: 2000, *Astron. Astrophys.* **355**, 365.
Godier, S. and Rozelot, M. J.: 1999, *Astron. Astrophys.* **350**, 310.
Gough, D. O.: 1982, *Nature.* **298**, 334.
Kosovichev, A. G.: 1996, *Astrophys. J.* **469**, 61.
Kuhn, J. R., Bush, R. I., Scheick, X. and Scherrer, P.: 1998, *Nature.* **392**, 155.
Lydon, T. J. and Sofia, S.: 1996, *Phys. Rev. Letters.* **76**, 177.
Morel, P.: 1997, *Astron. Astrophys.* **124**, 597.
Paternò, L., Sofia, S. and Di Mauro, M. P.: 1996, *Astron. Astrophys.* **314**, 940.
Pijpers, F. P.: 1998, *Monthly Notices Royal Astron. Soc.* **297**, 76.
Roxburgh, I. W.: 2001, *Astron. Astrophys.* **377**, 688.
Rozelot, J. P., Godier, S. and Lefebvre, S.: 2001, *Solar Phys.* **198**, 223.
Rozelot, J. P. and Rösch, J.: 1997, *Solar Phys.* **192**, 11.
Scherrer, P. H., et al.: 1995, *Solar Phys.* **162**, 129.
Snodgrass, H. B.: 1992, in ASP Conf. Serf. 27, The Solar Cycle, ed. K. L. Harvey (San Francisco : ASP), 205.
Thuillier, G., Joukoff, A. and Schmutz, W.: 2003, *Proc. ISCS 2003 Symposium, Solar Variability as an Input to the Earth's Environment*, Tatransk Lomnica, Slovakia, 23-28 june 2003 (ESA SP-535, September 2003)
Ulrich, R. K. and Hawkins, G. W.: 1981a, *Astrophys. J.* **246**, 985.
Ulrich, R. K. and Hawkins, G. W. "Erratum": 1981b, *Astrophys. J.* **249**, 831.